

Improved Geometrical Scaling at the LHC

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We show that geometrical scaling exhibited by the p_T spectra measured by the CMS collaboration at the LHC is substantially improved if the exponent λ of the saturation scale depends on p_T . This dependence is shown to be the same as the dependence of small x exponent of F_2 structure function in deep inelastic scattering taken at the scale $p_T \simeq Q/2$.

Recently in Refs.[1, 2] it has been shown that p_T spectra measured by the CMS collaboration [3] at the LHC exhibit geometrical scaling. Geometrical scaling was first introduced in the context of Golec-Biernat–Wüsthoff (GBW) model [4] of deep inelastic scattering (DIS) in Ref.[5]. There, a reduced γ^* -proton cross-section $\sigma_{\gamma^*p}(Q^2, x)$ that in principle depends on two independent kinematical variables: Q^2 and Bjorken x , for small x (*i.e.* $x < 0.01$ or so) does depend effectively only upon the ratio

$$\tau = Q^2/Q_{\text{sat}}^2(x). \quad (1)$$

Here $Q_{\text{sat}}(x)$ is so called saturation momentum which is proportional to the transverse gluon density [4]

$$xg(x, Q^2) \sim \frac{\sigma_0}{\alpha_s(Q_{\text{sat}}^2)} Q_{\text{sat}}^2(x) \quad (2)$$

with σ_0 being dimensional constant that in the GBW model is equal to 23 mb. Since gluon density rises for small x like a power, saturation momentum is customarily assumed to take the following form:

$$Q_{\text{sat}}^2(x) = Q_0^2 (x/x_0)^{-\lambda} \quad (3)$$

with constant $\lambda = 0.2 \div 0.3$, determined from the HERA the data.

This new energy scale emerges in the models that incorporate gluon saturation [6, 7], like *e.g.* color glass condensate [8, 9], although geometrical scaling itself is more general, and does not require saturation (*i.e.* gluon density may grow like a power for arbitrarily small x and the dipole-proton cross-section needs not to go to a constant for large dipole sizes). Nevertheless, the existence of the saturation scale in strong interactions is by now well established. If so, it should also manifest itself in hadronic collisions, and it indeed does, as it was shown in Refs.[1, 2] where a simple Ansatz, based on dimensional analysis, for the saturation momentum has been proposed:

$$Q_{\text{sat}}^2 = Q_0^2 \left(\frac{p_T}{W} \right)^{-\lambda}. \quad (4)$$

Here $W \sim \sqrt{s}$ and $Q_0 \sim 1$ GeV sets the scale. It turns out that the charged particle p_T spectra measured at the

LHC at three incident energies: 0.9, 2.36 and 7 TeV exhibit geometrical scaling, *i.e.* they fall on one energy-independent universal curve $F(\tau)$

$$\frac{dN_{\text{ch}}}{dydp_T^2} = \frac{1}{Q_0^2} F(\tau) \quad (5)$$

if plotted in terms of the scaling variable

$$\tau = p_T^2/Q_{\text{sat}}^2 \quad (6)$$

with $\lambda \sim 0.27$. In essence, geometrical scaling for the p_T spectra (5) boils down to the prescription that allows to relate multiplicity distributions at two different energies W and W' . If

$$\frac{dN_{\text{ch}}}{d\eta d^2p_T}(p_T, W) = \frac{dN_{\text{ch}}}{d\eta d^2p'_T}(p'_T, W') \quad (7)$$

then the transverse momenta at which Eq.(7) holds, satisfy

$$p'_T = p_T \left(\frac{W'}{W} \right)^{\lambda/(\lambda+2)}. \quad (8)$$

This formula is independent of Q_0 and of the overall energy scale of W or W' . So the only relevant parameter of geometrical scaling is exponent λ .

Equations (7) and (8) allow to *rescale* p_T of known spectrum at energy W to another energy W' , and *predict* p_T spectrum at this energy, provided we know the value of λ . In the following, transverse momentum spectra obtained that way will be referred to as *rescaled* spectra.

Alternatively, if we do not know λ but we know spectrum at W , we can find λ by changing its value until equality (7) is satisfied, *i.e.* until the rescaled and true spectra coincide within errors.

In dipole models of DIS the quality of phenomenological fits is further increased provided one incorporates DGLAP Q^2 dependence [10] of the saturation scale (2). Furthermore, since one "measures" Q_{sat} with a Q^2 dependent probe (*e.g.* with virtual photon or a p_T hadron), effective saturation scale $Q_{\text{sat,eff}}$ acquires some dependence on the virtuality of the probe. These two effects may be conveniently accounted for by replacing $\lambda \rightarrow \lambda(Q^2)$. Indeed, for large Q^2 DIS structure function F_2 that is directly related to the saturation scale [4] behaves as:

$$F_2(x, Q^2) \sim \sigma_0 Q_{\text{sat,eff}}^2 \sim \frac{1}{x^{\lambda_{\text{eff}}(Q)}}. \quad (9)$$

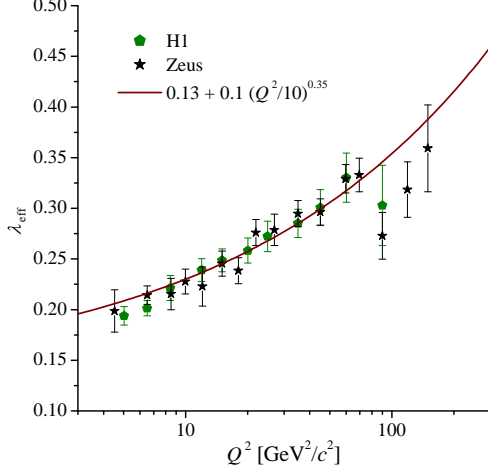


FIG. 1. Dependence of λ_{eff} on Q^2 from HERA (HERA data points [11] after Ref.[12]).

Power $\lambda_{\text{eff}}(Q)$ has been extracted from the HERA data [11] (see *e.g.* recent Ref.[12] and references therein) as shown in Fig. 1. In the same figure we plot an eyeball fit to the experimental points given by a simple function

$$\lambda_{\text{eff}}(Q) = 0.13 + 0.1 \left(\frac{Q^2}{10} \right)^{0.35}. \quad (10)$$

An interesting question arises, whether exponent λ that governs geometrical scaling in hadronic collisions exhibits any p_T dependence and, if yes, whether it is similar to the one obtained in DIS. For p_T -dependent λ formula (8) takes the following form:

$$p_T^2 \left(\frac{p_T}{W} \right)^{\lambda(p_T)} = p_T'^2 \left(\frac{p_T'}{W'} \right)^{\lambda(p_T')}. \quad (11)$$

A simple way to calculate approximate p_T dependence of λ is to use Eq.(8) bin by bin in p_T instead of an exact equation (11). For slowly varying $\lambda(p_T)$ such a procedure should give a good first order approximation. To this end we choose to rescale transverse momenta of CMS multiplicity spectra at $W = 0.9$ and 7 TeV to the reference energy $W' = 2.36$ TeV for some initial value of λ . Next, we compare the rescaled spectra with the experimental data at W' and repeat the whole procedure until the rescaled and true spectra coincide (7). In that way we obtain $\lambda(p_T)$. Since in general for W' there is no data point at p_T' obtained from (8), we have to interpolate the reference spectrum and its errors (in the following we neglect interpolation errors). The result of this interpolation is depicted in Fig. 3 by a grey band. In order to estimate the error of $\lambda(p_T)$ we add in quadrature errors of the W spectrum and the interpolated error of the reference W'

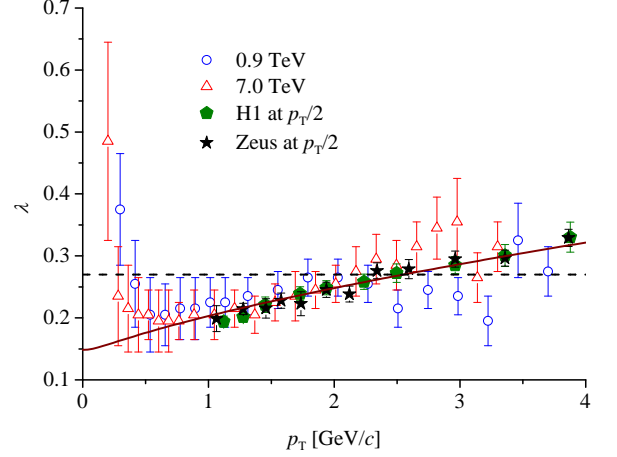


FIG. 2. Dependence of λ on p_T . Open circles correspond to λ obtained by rescaling 0.9 TeV data to the reference energy of 2.36 TeV, whereas open triangles correspond to 7 TeV. Exponent λ_{eff} extracted from HERA depicted by full pentagons (H1) and stars (Zeus) is plotted in function of $p_T = Q/2$ (see text). Solid line corresponds to Eq.(10) taken at $Q = 2p_T$.

spectrum at p_T and p_T' respectively. Calling this effective error ε^2 , we repeat the whole procedure solving equation

$$\frac{dN_{\text{ch}}}{d\eta d^2p_T}(p_T, W) - \frac{dN_{\text{ch}}}{d\eta d^2p_T'}(p_T', W') \pm \varepsilon = 0 \quad (12)$$

for $\lambda_{\pm} = \lambda \pm \delta$. The result is plotted in Fig.2 together with λ_{eff} from DIS taken at the scale $Q^2 \simeq 4p_T^2$. Both fit (10) and the data points are displayed. We see that indeed λ does depend on the transverse momentum. The agreement between λ extracted from the spectrum rescaled from $W = 0.9$ TeV (blue circles) and from $W = 7$ TeV (red triangles) is a signature of geometrical scaling. In an interval from 0.5 to approximately 2.5 GeV λ rises slowly with increasing p_T . Interestingly, p_T dependence of λ in this interval is in a surprising accordance with DIS $\lambda_{\text{eff}}(Q)$ taken at $p_T = Q/2$. For higher p_T data become too noisy to draw definite conclusions. The smooth behavior changes completely for $p_T < 0.5$ GeV where the steep rise of λ with decreasing p_T is seen. This may be a signal of an onset of a some other component in the production mechanism. Here, however, the assumption of slowly varying λ breaks down and more numerical care is needed before quantitative conclusions concerning small p_T part can be drawn. One should also stress at this point that further analysis of low p_T geometrical scaling requires good quality low momentum data.

Final conclusion that has to be drawn from Fig.2 is that geometrical scaling with constant λ is certainly a good first approximation, but a mild p_T dependence of λ improves substantially the quality of geometrical scaling.

This is depicted in Fig.3 where we plot the p_T spectra in terms of the rescaled momentum p'_T in the vicinity of 1 GeV where the difference between constant $\lambda = 0.27$ and "running" λ of Eq.(10) is most pronounced. Black points and the shaded band correspond to the CMS spectrum (and its interpolation) at 2.36 TeV. Blue and red points (connected by dashed lines) correspond to 0.9 and 7 TeV spectra respectively, rescaled to the reference energy of 2.36 TeV for constant λ and "running" $\lambda_{\text{eff}}(2p_T)$ of Eq.(10). An improvement for "running" λ is evident. For other p_T intervals where the difference between constant and "running" λ is not large, the quality of geometrical scaling is – obviously – comparable.

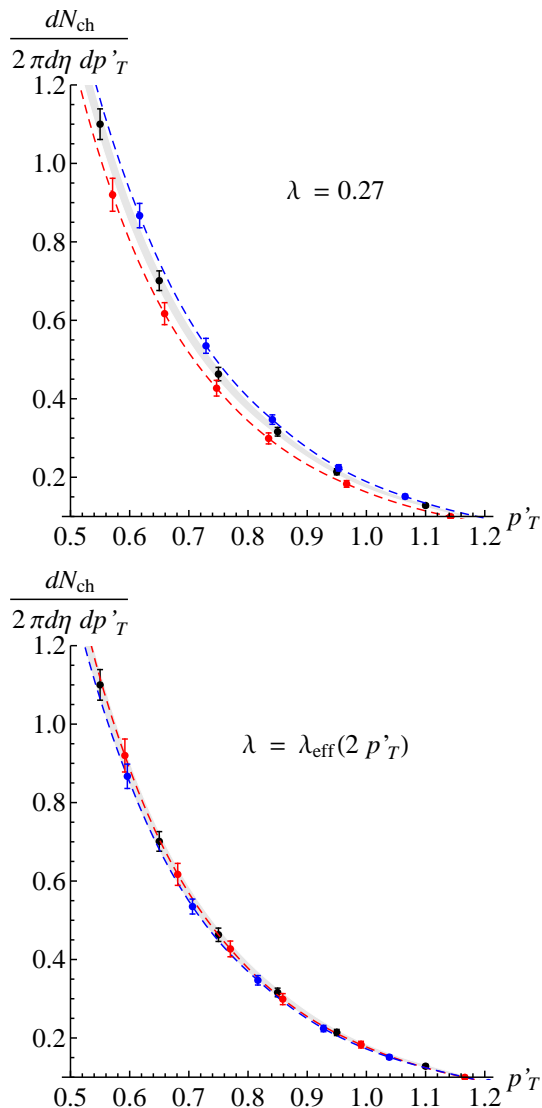


FIG. 3. Multiplicity density for $\sqrt{s} = 2.36$ TeV (black points and shaded band) as measured by CMS, and 0.9 TeV (blue points and dashed line) and 7 TeV (red points and dashed line) spectra rescaled to 2.36 TeV using hypothesis of geometrical scaling with constant $\lambda = 0.27$ and "running" effective $\lambda_{\text{eff}}(2p'_T)$ for low p_T . Horizontal scale in GeV/c.

Geometrical scaling in hadronic collisions is by far less obvious than in DIS. In DIS we have at our disposal simple theoretical (GBW) model [4] that allows to identify kinematical variables relevant for geometrical scaling. In hadronic collisions such models exist [13–17] but they rely on k_T factorization which has not been proven for soft particle production in central rapidity. Nevertheless, if k_T factorization is assumed, like in the recent studies of Refs.[16, 17], then the proportionality of multiplicity of produced gluons to the saturation momentum, and therefore geometrical scaling – assuming local parton-hadron duality – can be derived in a rather straightforward way (see *e.g.* [14]). Nevertheless, the exact form of the scaling variable τ , that in principle may depend also on rapidity, is to some extent a matter of educated guess. Luckily, for constant λ some uncertainties cancel out in Eq.(8), showing that the only relevant parameter is exponent λ .

Another notable difference between DIS and hadronic collisions is that in DIS we deal with totally inclusive cross-section, whereas in pp both hadronization and final state interactions play essential role. Nevertheless the imprint of the saturation scale Q_{sat} is visible in the spectra, which means that the information on the initial fireball survives until final hadrons are formed.

In this letter we have shown that the quality of geometrical scaling improves if the exponent λ becomes p_T -dependent. We have computed this dependence by rescaling p_T spectra at 0.9 and 7 TeV to the reference energy 2.36 TeV, however we have also checked that rescaling 0.9 and 2.36 TeV spectra to 7 TeV or 7 and 2.36 TeV spectra to 0.9 TeV gives qualitatively the same results. Not only p_T spectra rescaled from different energies to the reference energy W' agree (which is the essence of geometrical scaling), but the p_T dependence of the exponent λ agrees with the dependence obtained from DIS $\lambda_{\text{eff}}(Q)$, at the scale $Q \sim 2p_T$. We find this last result remarkable, since it provides a direct link between two different types of reactions.

Several points require further clarification. First of all new large p_T data of good quality will be of importance to test the range of applicability of geometrical scaling and of the discussed similarity with DIS. Also low p_T data, where hadronic $\lambda(p_T)$ deviates from the one from DIS, is required to see whether this deviation signals an onset of a new production mechanism *common* for different energies, or whether different energies require different $\lambda(p_T)$ violating geometrical scaling in this region. It will be interesting to verify if geometrical scaling works also in heavy ion collisions. If so, p_T spectra in heavy ion collisions measured at different energies and at different centralities will allow find A dependence and impact parameter dependence of Q_{sat} .

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